

# Finite-Element Formulation for Dynamic Strain Localization and Damage Evolution in Metals

Hashem M. Mourad, Curt A. Bronkhorst,  
Thomas R. Canfield, Francis L. Addessio, T-3

Due to their inability to represent strain localization correctly, conventional computational methods yield mesh-dependent results in ductile failure problems. In the computational framework presented here, localization behavior is predicted using a material stability analysis and localized deformation modes are represented accurately and efficiently by embedding localization bands within larger computational elements. This framework also allows the use of different constitutive models inside and outside the localization band.

The formation of strain localization bands is often observed in metals undergoing high-rate plastic deformation. The material in these narrow bands undergoes intense plastic straining. In many cases strain localization is caused by—and subsequently interacts with—material softening mechanisms (e.g., void nucleation and growth, thermal softening), leading ultimately to failure. Thus, the effective treatment of ductile failure problems in a computational setting requires that localized deformation modes be represented accurately.

With conventional finite element techniques, this can only be achieved by resolving localization bands explicitly, that is, via mesh refinement, which is prohibitively expensive given the small width of a typical localization band (10–30  $\mu\text{m}$ ) compared to the characteristic dimension of a specimen ( $\sim 10$  mm) or structure ( $\sim 1$  m). In addition, conventional methods are known to yield mesh-dependent results, for example, predicted localization bands tend to follow mesh lines.

To circumvent these difficulties, an explicit finite-element formulation for dynamic strain localization was developed, based on the assumed-strain technique of Belytschko et al. [1]. In this formulation, a material stability analysis is used to check for incipient localization behavior at each material (Gauss quadrature) point. Notably, the location and orientation of the nascent localization band are also determined by the same stability analysis, instead of being dictated by the dimensions or orientation of the mesh.

In the absence of localization, the strain field within a given element remains continuous. Once the onset of localization is detected, however, the discontinuous strain mode associated with the nascent localization

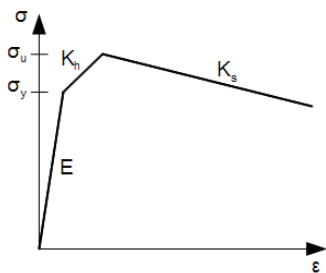
band is added to the element's strain field and is allowed to evolve gradually. This approach ensures a smooth transition from uniform to localized deformation. In addition, embedding localization bands in this manner, within significantly larger computational elements, allows spatially converged solutions to be obtained at a reasonable computational cost. It is also important to note that the width of the localization band is treated as a material parameter, completely independent of the mesh size.

As a numerical example, we consider a rectangular block (length = 0.14 m, width = 0.1 m) subjected to plane strain extension. The material is assumed to follow the elasto-plastic material law illustrated in Fig. 1, and has the following material properties: Young's modulus  $E = 200$  GPa, Poisson's ratio  $\nu = 0.29$ , isotropic hardening/softening moduli  $K_h = 220$  MPa,  $K_s = -66$  MPa (with a negative value signifying softening), yield strength  $\sigma_y = 300$  MPa, ultimate strength  $\sigma_u = 310$  MPa, and mass density  $\rho = 7850$  kg/m<sup>3</sup>.

Exploiting symmetry to reduce computational cost, only the upper right quadrant of the block is modeled. An upward velocity of 2 m/s is applied to the upper boundary, as shown in Fig. 2. The localization band is assumed to have width  $b = 3.33$  mm. The material strength is reduced in the bottom-left element ( $\sigma_y = 200$  MPa,  $\sigma_u = 210$  MPa) to simulate the existence of a material imperfection, making this element a favorable nucleation site for localization bands.

To examine the ability of the formulation in alleviating mesh dependency, the problem is first solved using a coarse mesh consisting of  $5 \times 7$  elements, and then repeated using a mesh including  $10 \times 14$  elements.

Fig. 1. Illustrated elasto-plastic material law.



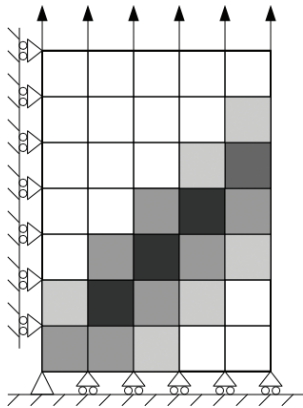


Fig. 2. Exploiting symmetry to reduce computational cost, only the upper right quadrant of the block is modeled. An upward velocity of 2 m/s is applied to the upper boundary. Shaded area represents the localization band that forms as the block deforms. Elements where the material stability analysis indicates that localization behavior was initiated at some stage of the loading process, at all (four) Gauss points, are shown in the darkest shade. Lighter shades signify instability at a smaller number of Gauss points.

The results of these two simulations are then compared to the reference solution that is obtained by explicitly resolving the localization band using a fine mesh consisting of  $15 \times 21$  conventional elements.

The load-deflection curve and the deformed shape of the block, as predicted by each of the three meshes, are shown in Figs. 3 and 4, respectively. It is clear from Fig. 3 that the formulation is successful in capturing the correct response of the overall structure throughout the loading process, including the softening regime. It can also be seen from Fig. 4 that the location and orientation of the localization band, and its effect on the overall deformation of the structure, are captured reasonably well, even by the coarsest mesh.

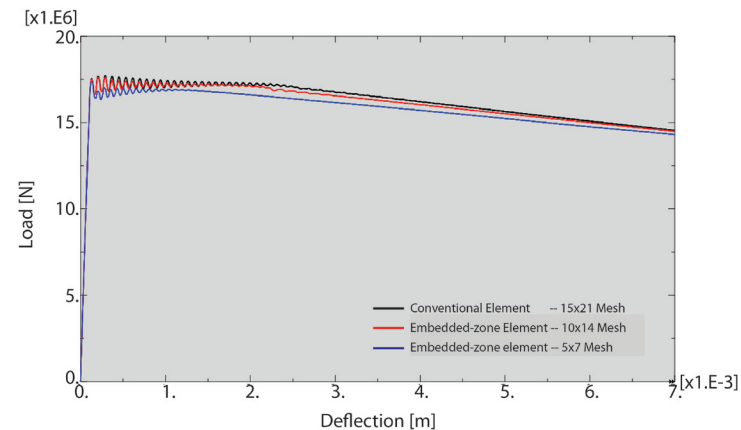
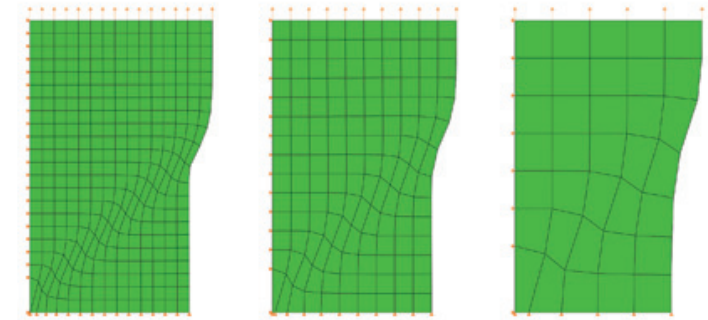


Fig. 3. The load-deflection curve, as predicted by each of the three meshes.



Step: Step-1  
Increment 400568: Step Time= 4.0000E-03  
Deformed Var: U Deformation Scale Factor: +1.000e+00

Fig. 4. The deformed shape of the block, as predicted by each of the three meshes.

[1] Belytschko, T. et al., *Comput Meth Appl Mech Eng* **70**, 59 (1988).

#### Funding Acknowledgment

DOE/DoD Joint Munitions Program; LANL Laboratory Directed Research and Development Program